# Calculation Policy

# About our Calculation Policy

The following calculation policy has been written to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. It is designed to give children a clear and consistent progression of learning calculations across school. Mathematics in Nursery and Reception follow the 'Development Matters' EYFS document; this policy builds on the content and methods established in the Early Years Foundation Stage.

The calculation policy is organised into age stage expectations as set out in the National Curriculum 2014. It is vital that children are taught according to the stage they are currently working at. They can then be moved onto the next level as soon as they are ready or work at a lower stage until they are secure enough to move on.

This document is intended to aid a consistent approach to teaching mental and written calculations within New Hall primary school, therefore establishing continuity and progression throughout the school.

As a school we value the involvement of parents, teaching assistants and other support staff as well as teachers in developing children's learning. It is vital therefore that everyone is familiar with the way we teach mathematics in order that our children can fulfil their potential in mathematics.

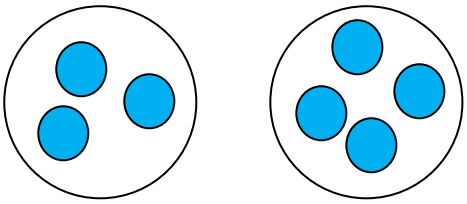
Children's understanding develops more quickly and securely when they experience calculating as two concepts rather than the four separate operations. Therefore this policy is set out with addition and subtraction together for each year group and multiplication and division together. In this way connections can be made easily.

When we teach children how to calculate they will experience the mathematics to develop a real number sense by working with concrete materials whenever possible. They will learn to record their work by representing their concrete experience as pictures, diagrams and symbols. In time, these symbols as a record of calculation, become something that children can use as an abstract 'device' for solving calculations. So what starts as a record of a concrete situation or event becomes a 'method' when it has been practised and reinforced many times over a period of time.

# CALCULATION IN STAGE 0

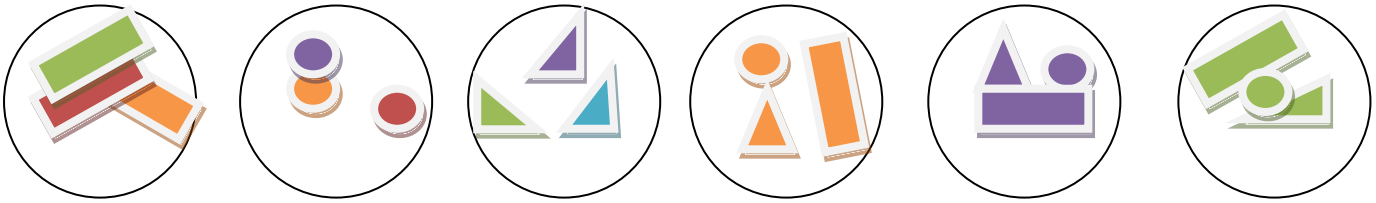
Children use Numicon to identify the value of the shapes and match this with numerals. They also find one more and one less and add and subtract by placing the Numicon pieces onto and next to each other. Children use a variety of ways to count for example clapping or counting steps. They also have access to a wide range of objects of different sizes to count.

Compare two sets of numbers



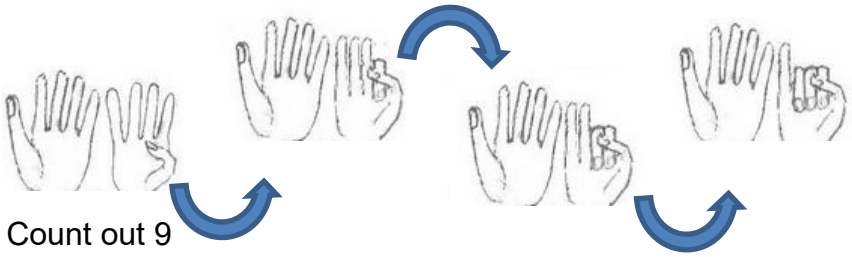
Separates a group of objects in different ways.

or



Children have a range of counting equipment. They find the total number of items in two groups by counting all of them. They use fingers, cubes and Numicon to support their calculations.

Children use the vocabulary involved in adding and subtracting and do this practically and with real life contexts. Children count up to find the difference and count all and count back to subtract.

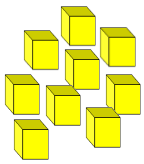


Count out 9 fingers

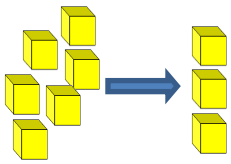
Fold down 3 fingers "…1, 2, 3.."

Count all of the remaining fingers "… 1, 2, 3, 4, 5, 6"

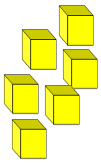
Children also



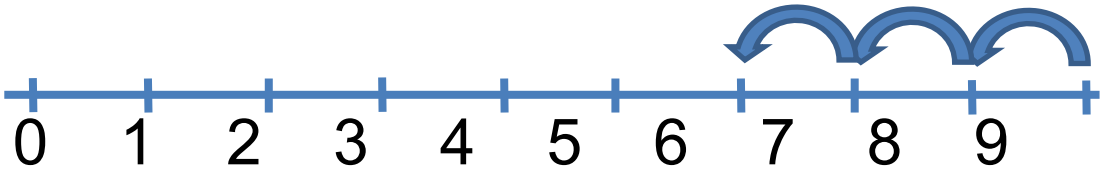
Count out 9 cubes



Move 3 cubes away "…1, 2, 3.."



Count all of the remaining cubes "… 1, 2, 3, 4, 5, 6"




Children count back on number lines and count the jumps.

Key Vocabulary:

number, order, count, pattern, a lot, more, few, less, same, next, one, two, three… How many..? Which/what number…? What does this number tell us?

more, less, few, more than, less than, fewer, fewer than, many, lots, lots of, a few, a lot

What number comes next? How many? What does the number look like?

Least, amount, the same as

How many altogether? total, one more, one less, estimate, guess, after

How many are left?

about, count on, count back, put together, add together, take away

Subitising

Composition

Number names to 30

Number Bonds

Subtract, difference, what is the difference between?

## Counting

The one-one principle

This involves children assigning one number name to each object that is being counted. Children need to ensure that they count each object only once ensuring they have counted every object.

The stable-order principle

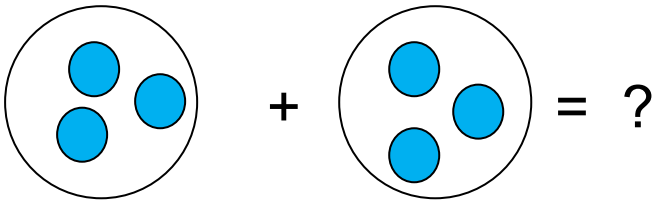
Children understand when counting, the numbers have to be said in a certain order.

The Cardinal Principle
Children understand that the number name assigned to the final object in a group is the total number of objects in that group.

The Abstraction Principle
This involves children understanding that anything can be counted including things that cannot be touched including sounds and movements eg: jumps.

The Order-Irrelevance Principle
This involves children understanding that the order we count a group of objects is irrelevant. There will still be the same number.

**Early Learning Goals**
Using quantities and objects they add two single digit numbers and count on to find the numbers.




Children use practical equipment then number lines to add, by counting on in ones. Children are encouraged to start with the larger number and count on. They solve problems involving doubling, halving and sharing.

**Number**
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.
Have a deep understanding of number to 10, including the composition of each number.
Subitise (recognise quantities without counting) up to 5

**Numerical Pattern**
Compare quantities up to 10 in different contexts, recognising when one quantity is greater than , less than or the same as the other quantity.
Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
Verbally count beyond 20, recognising the pattern of the counting system.

## NON-NEGOTIABLES FOR STAGE 0

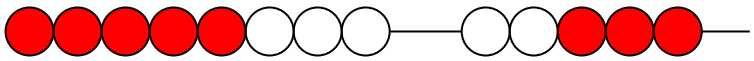
**Reception**
Count objects, actions and sounds
Subitise
Link the symbol (numeral) with its cardinal number value
Count beyond 10
Compare numbers
Understand the one more than/one less than relationship between consecutive numbers
Explore the composition of numbers to 10
Automatically recall number bonds for numbers 0-10

# Addition Stage One

**Add with numbers up to 20 and beyond**– conceptual understanding using concrete materials.

Use number lines to add, by counting on in ones. Encourage children to start with the larger number and count on. The use of Numicon is continued from Reception. The Numicon Shapes are used as a visual and concrete aid for addition. Large scale number lines are used for physical experience.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.
Using the example **8 + 5 =**



A number line is used to reinforce this concept.


To develop conceptual understanding children should:

- Have access to a wide range of counting equipment, and be encouraged to use it, they should use everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems using concrete objects and number line addition to solve them, e.g.

9 + 2 = ☐ 13 + 5 = ☐ 5 + 3 + 2 = ☐ ☐ + ☐ = 8 2 + 4 = 3 + 3

- Partition numbers in different ways.

| 18 | |
|---|---|
| 13 | 5 |

This builds on from prior learning of adding by combining two sets of objects into one group (e.g. 5 cubes and 3 cubes) in Stage 0.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Non-negotiables for addition in Stage One:

- Read and write whole number to 100 in numerals, including 1 – 20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple one step problems involving addition using objects, number lines and pictorial representations.

# Subtraction Stage One

**Subtract from numbers up to 20**

Children consolidate understanding of subtraction by using practical equipment such as bead strings and cubes. They are introduced to more formal recording using number lines.

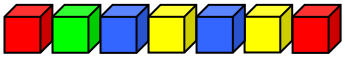
Children read, write and interpret number sentences with – and = signs.

Subtract by taking away; 8 - 4 = 4

Count back in ones on a number line to take away, with numbers up to 20.


We model subtraction using hundred squares, numbered numberlines and tracks and using practical resources such as Numicon.

Find the difference between:



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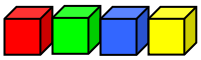


4

‘7 is 3 more than 4’
I am 3 years older
than my sister.’

This will be introduced practically as ‘find the difference between’ and ‘how many more?’ in a range of familiar contexts.

Children use a range of equipment to support their learning. Numicon can be used to reinforce the value of numbers to 20.

Mental subtraction:
Children should start recalling subtraction facts up to and within 10 and 20. They should be able to subtract zero.

**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _?

Non-negotiable for subtraction in Stage One:

- Given a number, say one more or one less.
- Count to and over 100, forward and back, from any number.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one digit and two digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes, Numicon) and pictures and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

# Multiplication Stage One

**Multiply with concrete objects, arrays and pictorial representations.**

If one Teddy has 2 legs, how many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag. How many sweets are in 5 bags altogether?.

3 + 3 + 3 + 3 + 3 = 15

Children should have experience of counting equal groups of objects in 2s, 5s and 10s.

Problem solving should be presented in a practical way involving counting equal groups or sets, as above.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count.

Non-negotiables for multiplication in Stage One:

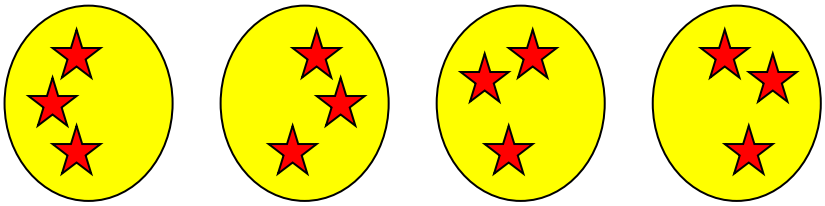
- Count in multiples of 2, 5 and 10.
- Solve one step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns and counting in 2s, 5s and 10s.
- Begin to understand doubling using concrete objects and pictorial representations.

# Division Stage One

**Group and share small quantities**
Using objects, diagrams and pictorial representations children solve problems involving both grouping and sharing.

**How many groups of 3 can be made with 12 stars?**

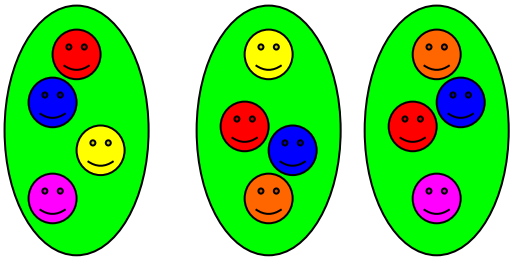
**Grouping**



There are 4 groups of 3.

**Can you share 12 sweets between 3 bowls?**

**Sharing**



4 4 4

12 shared between 3 is 4

Example division problem in a familiar context:
There are 6 pupils on a table and there are 18 pieces of fruit to share between them. If they share them equally, how many will each get?

Can they work it out and give a division statement…?
"18 shared between 6 people gives you 3 each".

Children should:
- Use lots of practical apparatus, arrays and picture representations.
- Be taught to understand the difference between 'grouping' objects (How many groups of two can you make?) and 'sharing' (share these sweets between two people).
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into two equal groups.

**Key vocabulary:** share, share equally, one each, two each … group, groups of, lots of, array.

Non-negotiables for division in Stage One:
- Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand division, finding simple fractions of objects, numbers and quantities.
- Children must make connections between arrays, number patterns and counting in 2s, 5s and 10s.

# Addition Stage Two

**Add with numbers – 2 digit numbers**

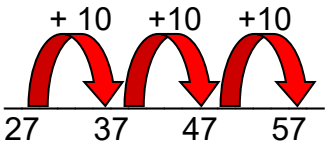
Children develop mental fluency with addition and place value involving 2 digit numbers, then establish more formal methods.


Use empty number lines, Numicon, concrete equipment, hundred squares etc to build confidence and fluency in mental addition skills.

**Step 1**

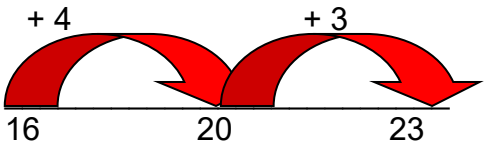
Add 2 digit numbers and tens:

27 + 30



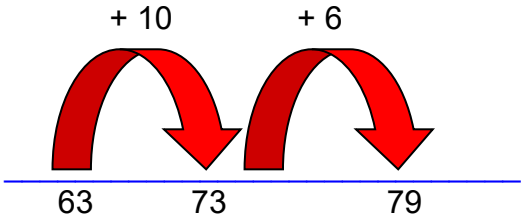

Add 2 digit numbers and units:

16 + 7




Add pairs of 2 digit numbers. Moving to the partitioned column method when secure adding tens and units.

63 + 16




23 + 34

$$\begin{array}{r} 20 + 3 \\ 30 + 4 \\ \hline 50 + 7 \\ =57 \end{array}$$

**Step 2**: Once children can add a multiple of 10 to a 2 digit number mentally (e.g. 80+11), they are ready for adding pairs of 2 digit numbers that DO cross the tens boundary (e.g. 58+43).

$$\begin{array}{r} 50 + 8 \\ +40 + 3 \\ \hline 90 + 11 \\ = 101 \end{array}$$

Children who are confident and accurate at this stage should continue to use the partitioned column method with 2 and 3 – digit numbers.

To support understanding, children physically make and carry out the calculation with Base 10 equipment, Numicon or place value counters, then compare their practical version to the written form, to help them build an understanding of it.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary.

Non-negotiables for addition at Stage Two:

- Add a 2 digit number and ones (e.g. 25 + 6)
- Add a 2 digit number and tens (e.g. 23 + 50)
- Add pairs of 2 digit numbers and tens (e.g. 32 + 46)
- Add three single digit numbers (e.g. 4 + 9 + 5)
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of 10 to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2 digit numbers (tens and ones)
- Compare and order numbers to 100 using < and > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

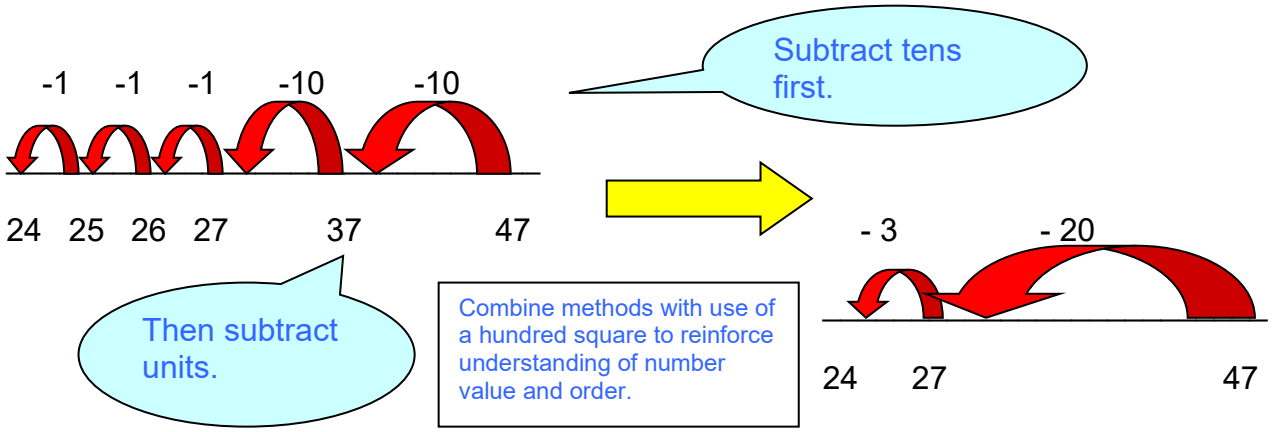
# Subtraction Stage Two

**Subtract with 2 digit numbers**

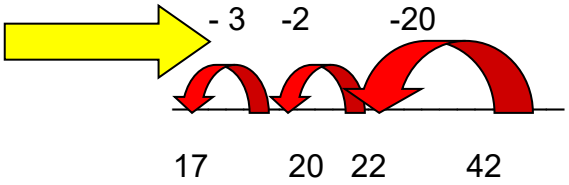
Subtract on a number line by counting back, aiming to develop mental subtraction skills.
This strategy will be used for:

- 2 digit numbers – subtract units by taking away/counting back, e.g. 38 – 7
- 2 digit numbers – subtract tens by taking away/counting back, e.g. 65 – 20
- Subtracting pairs of 2 digit numbers.

47 – 23 = 24 Partition the second number and subtract it in tens and units, as below:




Teaching children to bridge through ten can help them to become more efficient. For example 42-25:

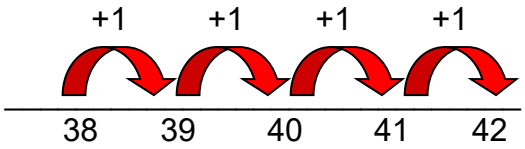



Children use Base 10, Numicon and counters to support their calculations.
Mental strategy – subtract numbers close together by counting on:

42 – 38 = 4

Start with the smaller number and count on to the largest.

Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction.




**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _? Difference, count on, strategy, partition, tens, units.

Non-negotiables for subtraction in Year Two:

- Recognise the place value of each digit in a two digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two digit number and ones, a two digit number and tens, and two digit numbers.
- Show that subtraction cannot be done in any order.
- Recognise and use the inverse relationship between addition and subtraction, using this to check calculations and missing numbers problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and words.

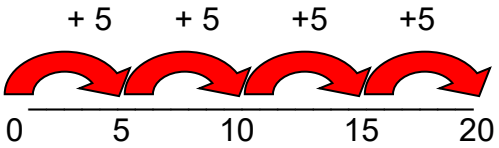
# Multiplication Stage Two

**Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)**

**Use repeated addition on a number line**

**4 x 5 = …**

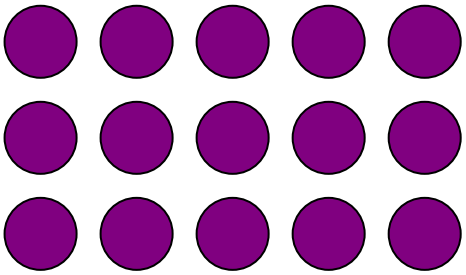
Starting from zero, make equal jumps on
a number line to work out multiplication
facts and write multiplication statements
using x and = signs.




**4 x 5 = 20**

**Use arrays**:

5 x 3 = 15



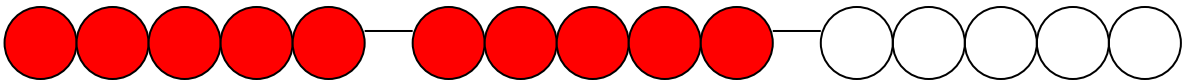
3 x 5 = 15

5 x 3 = 3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 5 + 5 +5 = 15

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x ____ = 6

**Use practical apparatus**:



**Use mental recall:**

Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times …

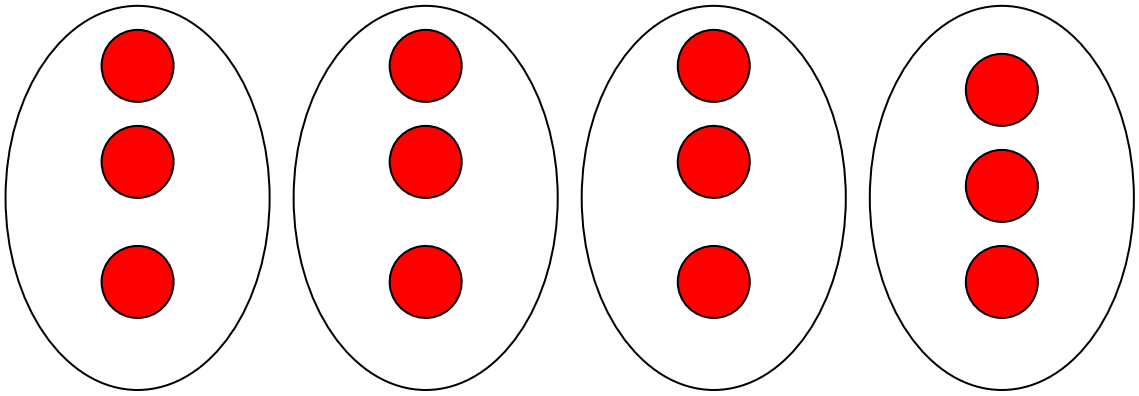
Non-negotiables for multiplication in Stage Two:

- Count in steps of 2, 3 and 5 from zero and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order (commutative)
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

# Division Stage Two

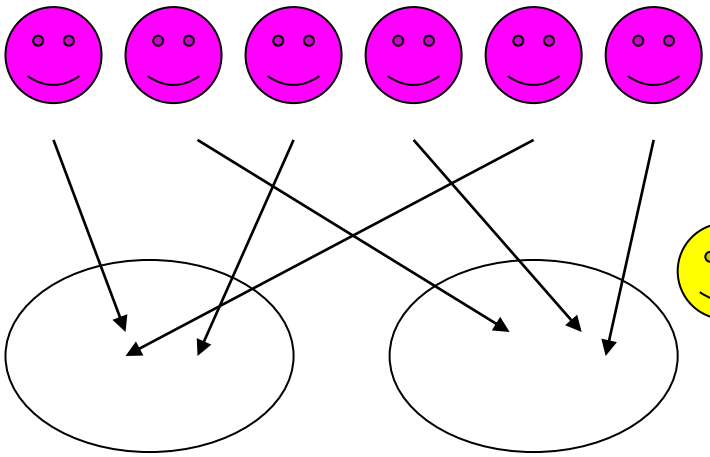
**Group and share using the ÷ and = signs**
Use objects, arrays, diagrams and pictorial representations and grouping on a number line.

**Arrays**: 12 ÷ 3 = 4 This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

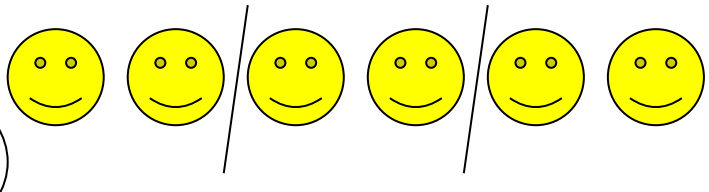


Children should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

**Know and understand sharing and grouping:**
6 sweets shared between 2 people, how many do they each get?



There are 6 sweets, how many people can have 2 sweets each?



**Children should be taught to recognise whether problems require sharing or grouping.**
Grouping using a number line: group from zero in equal jumps of the divisor to find out 'how many groups of __ in ___? Children could use a bead string or practical apparatus to work out problems like ' a book costs £3.00. How many books can I buy with £12.00? This is an important method to develop understanding of division as grouping.

**Key vocabulary:** share, share equally, one each, two each … group, groups of, lots of, array, divide, divide by, divided into, division, grouping, number line, left, left over.

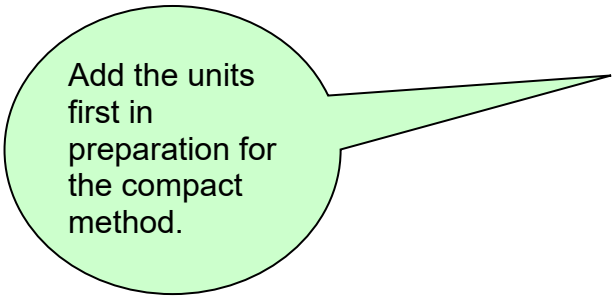
Key number skills needed for division in Stage Two:

- Count in steps of 2, 3 and 5 from 0.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division using arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts.

# Addition Stage Three

**Add numbers with up to 3 digits**

Continue the **expanded column addition** method:

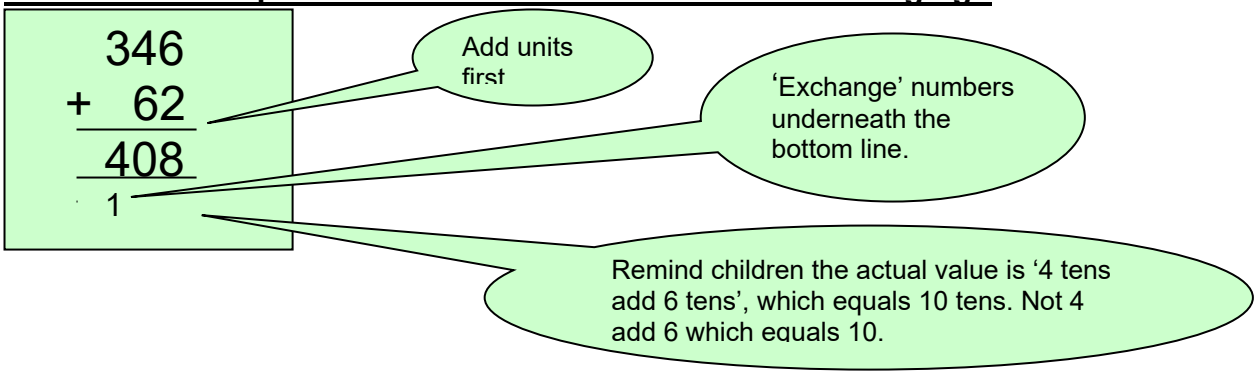



```
  346
+  62
   8
 100
 300
 408
```

**In order to carry out this method of addition:**

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Children need to be able to record in columns.

**Move to the compact column addition method with 'exchanging':**




- Children who are very secure and confident with 3 digit expanded column addition should be moved to the compact column addition method, being introduced to 'exchanging'.
- Children should compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'exchange', expanded, compact.

Non-Negotiables for addition in Stage Three:

- Read and write numbers to 1000 in numerals and words.
- Add two digit numbers mentally including those exceeding 100.
- Add a three digit number and ones mentally (175 + 8)
- Add a three digit number and ten mentally (345 + 40)
- Add a three digit number and hundreds mentally (468 + 300)
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing numbers problems, using number facts, place value and more complex addition.
- Recognise the place value of each digit in three digit numbers (hundreds, tens, ones)
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding to the nearest multiple of 10, 100, 1000 and adjusting, using near doubles, partitioning and recombining.

# Subtraction Stage Three

**Subtracting with two and three digit numbers**

Introduction of partitioned column subtraction method.

**Step 1**: introduce this method with examples where no exchanging is required.

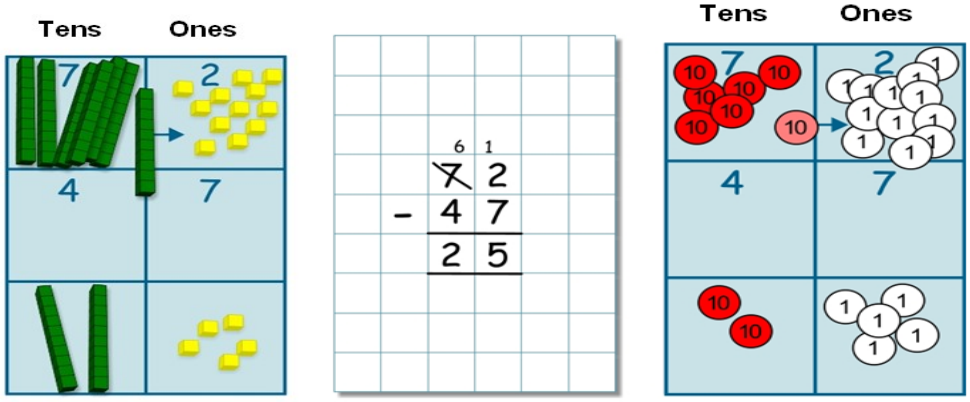
89 – 35 = 54

80 + 9

- 30 + 5

50 + 4

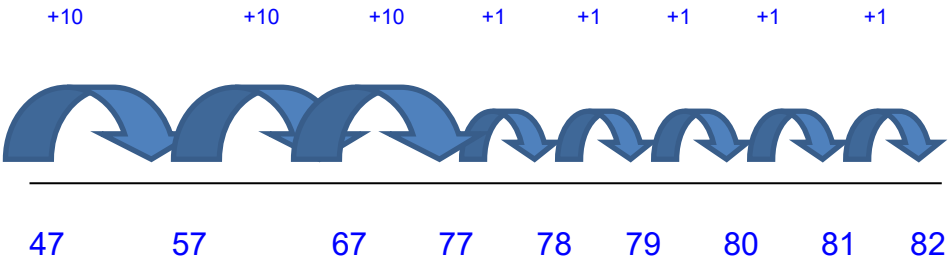
**Step 2**: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10 or place value counters then subtract the smaller number from it.




**Step 3:** once children are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2 and 3 digit numbers.

**Counting on as a mental strategy for subtraction:**

Continue to reinforce counting on as a strategy for close together numbers (e.g. 121 – 118 and also for numbers that are 'nearly' multiples of 10, 100, 1000 or £2, which make it easier to count on (e.g. 102 – 89, 131 – 79, or calculating change from £1 etc.)




Because counting in 10s is the way weuse a 100 square.

**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds,value, digit.

Non-Negotiables for subtraction in Stage Three:

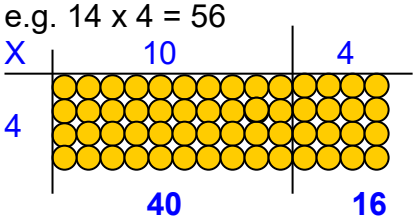
- Subtract mentally; three digit number and ones, three digit number and tens, three digit number and hundreds.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise place value of each digit in a three digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above).
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21) and selecting the most appropriate methods to subtract, explaining why.

# Multiplication Stage Three

## Multiply two digits by a single digit number

Introduce the grid method for multiplying 2 digit by single digits:
Link the layout of the grid to an array initially.

e.g. 14 x 4 = 56




e.g. 23 x 8 = 184

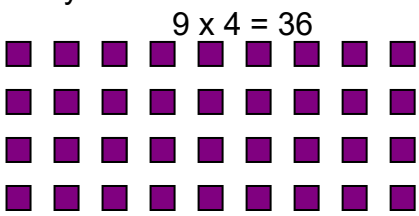
| x | 20 | 3 |
|---|---|---|
| 8 | 160 | 24 |

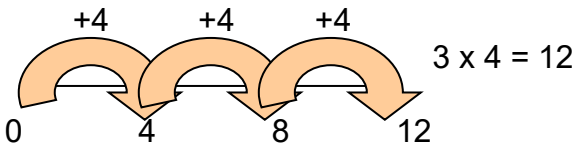
**160 + 24 = 184**

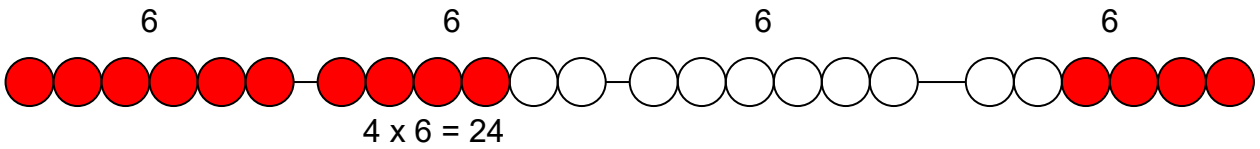
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 7 lots of 19 with 10s and 1s place value counters), then translate this to the grid method format.

To do this children must be able to:

- Partition numbers into tens and units.
- Multiply multiples of 10 by a single digit (e.g. 20 x 4 using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not know by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:




**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times …, partition, grid method, multiple, product, tens, units, value.

Non-Negotiables for multiplication in Stage Three:

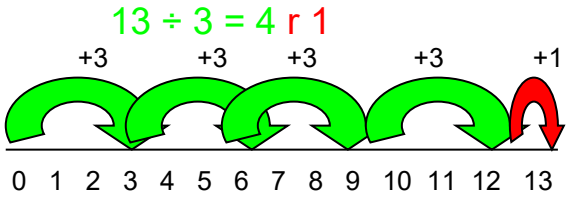
- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables and multiple multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including two digit x single digit, drawing upon mental methods and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems, e.g. using commutativity and for missing number problems.

# Division Stage Three

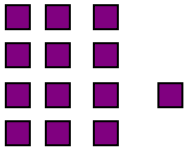
**Divide two digit numbers by a single digit (where there is a remainder in the final answer)**

**Grouping on a number line**:

Step 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s, and 10s, ready for 'exchanging' remainders across within the short division method.




13 ÷ 3 = 4 r 1



**Short division:** Limit numbers to no remainders in the answer.

Step 2: Once children are secure with division as grouping and sharing, demonstrate this using number lines, arrays etc. Short division for larger 2 digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders. Start by introducing the layout of short division by comparing it to an array. Children should reconstruct arrays and record what they have done as short division.

$$3\overline{)96}\quad 32$$

Remind children of correct place value, that 96 is equal to 90 but in short division pose:

- How many 3's in 9? = 3, record it above the 9 tens.
- How many 3's in 6? = 2, and record it above the 6 units.

**Short division:** Limit numbers to no remainders in the answer, but with remainders occurring within the calculation.

Step 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96 ÷4), and be taught to 'exchange' the remainder onto the next digit. If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

$$4\overline{)7^{3}2}\quad 18$$

Step 3 must only be taught when children can calculate with remainders.

Real life contexts need to be used routinely to help children gain a full understanding and the ability to recognise the place of division and how to apply it to problems.

**Key vocabulary:** share, share equally, one each, two each … group, groups of, lots of , array, divide, divide by, divided into, division, grouping, number line, left, left over, inverse, short division, 'exchange', remainder, multiple .

Non negotiables for division in Stage Three:

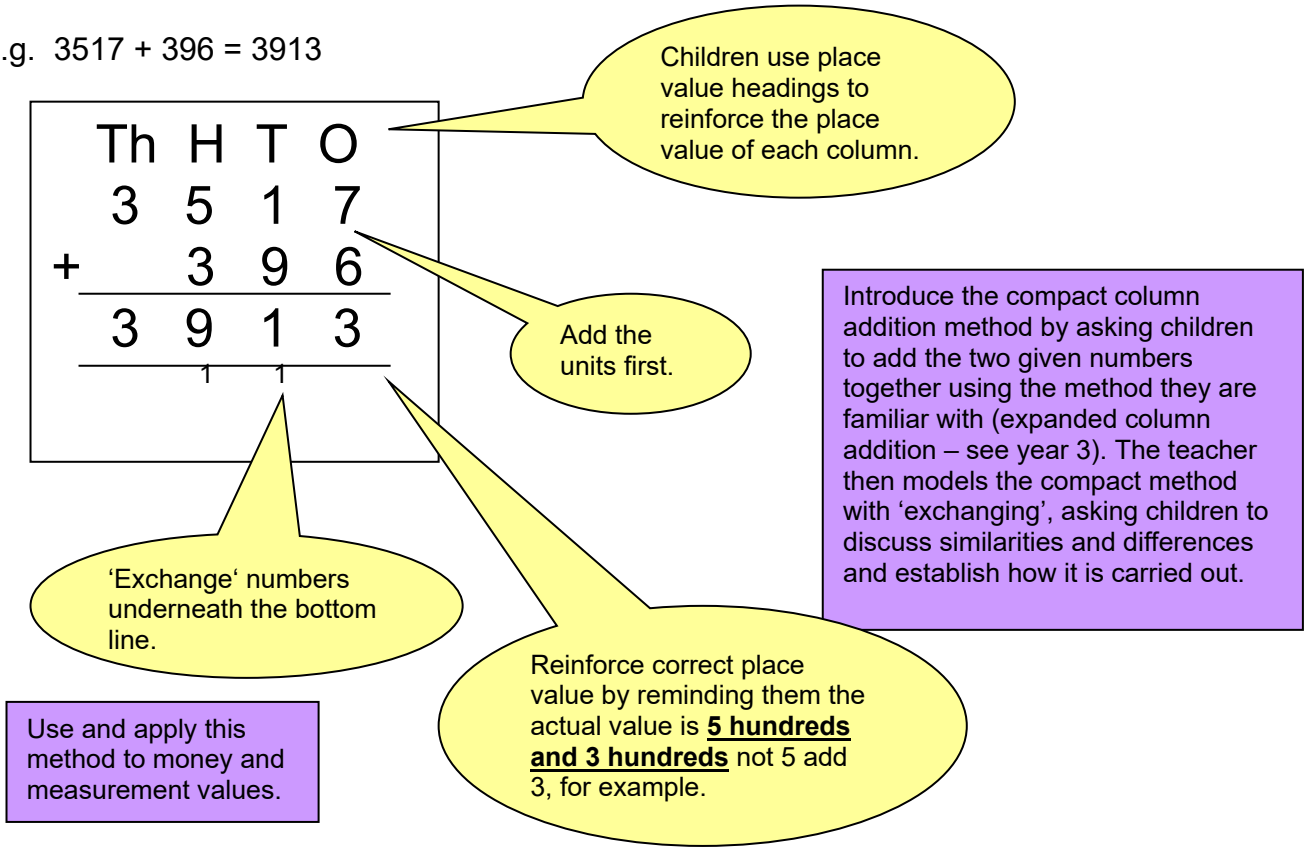
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2s, 4s and 8s)
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two digit numbers multiplied by one digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Children develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 x 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷3) to derive related facts (30 x 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Children develop a reliable written method for division, starting with calculations of two digit numbers by one digit numbers and progressing to the formal written method of short division.

# Addition Stage Four

**Add numbers with up to four digits**

Move from expanded addition to the compact column method, adding units first and 'exchanging' numbers underneath the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913




**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'exchange', expanded, compact, thousands, hundreds, digits, inverse.

Non-Negotiables for addition in Stage Four:

- Select most appropriate method: mental, jottings or written, and explain why.
- Recognise the place value of each digit in a four digit number.
- Round any number to the nearest 10, 100, 1000.
- Estimate and use inverse operations to check answers.
- Solve two step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with four digits using the formal method of column addition.
- Solve two step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

# Subtraction Stage Four

**Subtract with up to four digit numbers**

Partitioned column subtraction with 'exchanging' (decomposition):

2754 – 1562 = 1192

2000 + 700 + 50 + 4
- 1000 + 500 + 60 + 2

1000 + 100 + 90 + 2

As introduced in year 3, but moving towards more complex numbers and values. Use place value counters to reinforce 'exchanging'.

Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction:

| | Th | H | T | O |
|---|---|---|---|---|
| | 2 | 7 | 5 | 4 |
| - | 1 | 5 | 6 | 2 |
| | 1 | 1 | 9 | 2 |

To introduce the compact method ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask children to consider how it relates to the method that they know, what is similar, what is different? This is to develop and understanding of it.

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved – mental, counting on, counting back or written method.

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _ ?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse.

Non-Negotiables for subtraction in Stage Four:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction two step problems, choosing which operations and methods to use and why.
- Solve simple measures and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a four digit number.
- Round any number to the nearest 10, 100, 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

# Multiplication Stage Four

**Multiply two and three digits by a single digit using all multiplication tables up to 12 x 12**

Developing the grid method: children continue to use arrays for multiplication alongside developing the grid method.

E.g. 136 x 5 = 680

| x | 100 | 30 | 6 |
|---|---|---|---|
| 5 | 500 | 150 | 30 |

500
150
+ 30
680

Encourage column addition to add accurately.

Move onto short multiplication (see year 5) if and when children are confident and accurate multiplying 2 and 3 digit numbers by a single digit this way, and are already confident in 'exchanging' for written addition.

Children should be able to:

- Approximate before they calculate and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer, e.g. 346 x 9 is approximately 350 x 10 = 3500
  Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single digit, using their multiplication table knowledge.
- Recall all times tables up to 12 x 12

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times … partition, grid method, multiple, product, sets of, inverse.

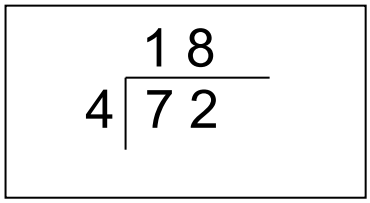
Non-Negotiables for multiplication in Stage Four:

- Count in multiples of 6, 7, 9, 25 and 1000.
- Recall multiplication facts for all multiplication tables up to 12 x 12.
- Recognise place value of digits in up to four digit numbers.
- Use place value, know facts and derived facts to multiply mentally, e.g. multiply 1, 10, 100, or by 0, or to multiply three numbers.
- Use commutativity and other strategies mentally, 3 x 6 = 6 x 3, 2 x 6 x 5 = 10 x 6, 39 x 7 = 30 x 7 + 9 x 7
- Solve problems with increasingly complex multiplication in a range of contexts.
- Recognise the place value of each digit in a four digit number (thousands, hundreds, tens and ones)

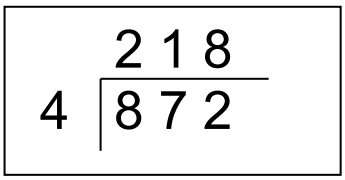
# Division Stage Four

**Divide up to three digit numbers by a single digit (without remainders initially).**

Continue to develop short division. However, it should be taught only when children have secured the skill of calculating with remainders and are have secure knowledge of place value. Arrays will still be used in conjunction with short division as the 'bus stop' is a representation of an array.

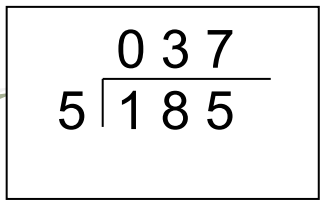



Step 1: children must be secure with the process of short division for dividing 2 digit numbers by a single digit (those that do not result in a final remainder), but must understand how to calculate remainders, using this to 'exchange' remainders within the calculation process (see example).




Step 2: children move onto dividing numbers with up to 3 digits by a single digit, however problems and calculations provided should not result in a final answer with a remainder at this stage. Children who exceed this expectation may progress to year 5 level.

Include money and measure contexts when confident.




When the answer for the first column is 0 (1 ÷ 5, as in the example), children could initially write a zero above to acknowledge its place, and must always 'exchange' the number (1) over to the next digit as a remainder.

Real life contexts need to be used routinely to help children gain a full understanding and the ability to recognise the place of division and how to apply it to problems.

**Key vocabulary:** share, share equally, one each, two each … group, groups of, lots of, array, divide, divide by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor.

Key number skills needed for division in Stage Four:

- Recall multiplication and division facts for all numbers up to 12 x 12
- Use place value, known and derived facts to multiply and divide mentally, including; multiplying and dividing by 10, 100 and 1.
- Children practise to become fluent in the formal written method of short division with exact answers when dividing by a one digit number.
- Pupils practise mental maths and extend this to three digit numbers to derive facts, for example 200 x 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

# Addition Stage Five

**Add numbers with more than four digits**

Including money, measures and decimals with different numbers of decimal places. Children should will use place value counters to model their calculations to show understanding.

The decimal point should be aligned in the same way as other place value columns, (aligned with the middle of the digits and on the line if using squared paper) and must be in the same column in the answer.

```
  £23·59
+ £7·55
  ------
  £31·14
   1 1  1
```

Numbers should exceed 4 digits.

```
  23481
+  1362
  ------
  24843
     1
```

Children should be able to add more than 2 values carefully aligning place value columns.

```
  19·01
   3·65
+  0·70
  ------
  23·36
   1 1
```

Say '6 tenths add 7 tenths' to reinforce place value.

Empty decimal places can be filled with a zero to show the place value in each column.

Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places. Numicon is used to reinforce the value of tenths. Decimal number lines and place value charts also support children's work with decimal numbers.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal place, decimal point, tenths, hundredths, thousandths.

Non-Negotiables for addition in Stage Five:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies, i.e. add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, inverse, partitioning and re-combining, using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to a least one million and determine the value of each digit.
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000.
- Add numbers with more than four digits using formal written method of columnar addition.

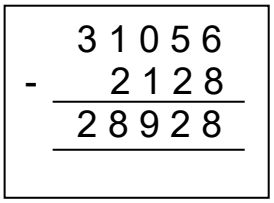
# Subtraction Stage Five

**Subtract with at least four digit numbers including money, measures, decimals.**

**Compact column subtraction (with 'exchanging').**

Create lots of opportunities for subtracting and finding differences with money and measures.

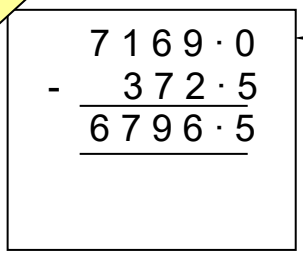
Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.




Subtracting with larger integers.

Children should be encouraged to estimate the answer first and then check their estimate against the answer.

Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.




Subtracting with decimal values, including mixtures of integers and decimals, aligning the decimal point.

**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _ ?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Non-Negotiables for subtraction in Stage Five:

- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least one million and determine the value of each digit.
- Count forwards and backwards in steps of powers of 10 for any given number up to one million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to one million to the nearest 10, 100, 1000, 10,000 and 100,000.

# Multiplication Stage Five

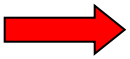
**Multiply up to four digits by one or two digits**

Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are fewer steps involved in the column method.
- Children need to be taught to approximate first, e.g. for 72 x 38 they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and use the approximation to check the reasonableness of their answer.

**Short multiplication for multiplying by a single digit:**

| x | 300 | 20 | 7 |
|---|---|---|---|
| 4 | 1200 | 80 | 28 |

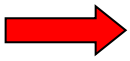


$$\begin{array}{r} 327 \\ \times \quad 4 \\ \hline 1308 \\ {\scriptstyle 1\,2} \end{array}$$

Children could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

**Introduce long multiplication for multiplying by 2 digits:**

| | 10 | 8 |
|---|---|---|
| 10 | 100 | 80 |
| 3 | 30 | 24 |



$$\begin{array}{r} 18 \\ 13 \\ \hline 54 \\ 180 \\ \hline 234 \end{array}$$

18 x 3 on the first row

(8 x 3 = 24, exchanging the 2 for twenty, '1' x 3).

18 x 10 on the second row. Put a zero in the units first, then say 8 x 1, and 1 x 1.

**Moving towards more complex numbers:**

The grid could be used to introduce long multiplication as the relationship can be seen in each row.

$$\begin{array}{rl} 1234 & \\ \times \quad 16 & \\ \hline 7404 & (1234 \times 6) \\ 12340 & (1234 \times 10) \\ \hline 19744 & \end{array}$$

Children should be taught to estimate the answer first then check their answer against their estimate.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times … partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, 'exchange'

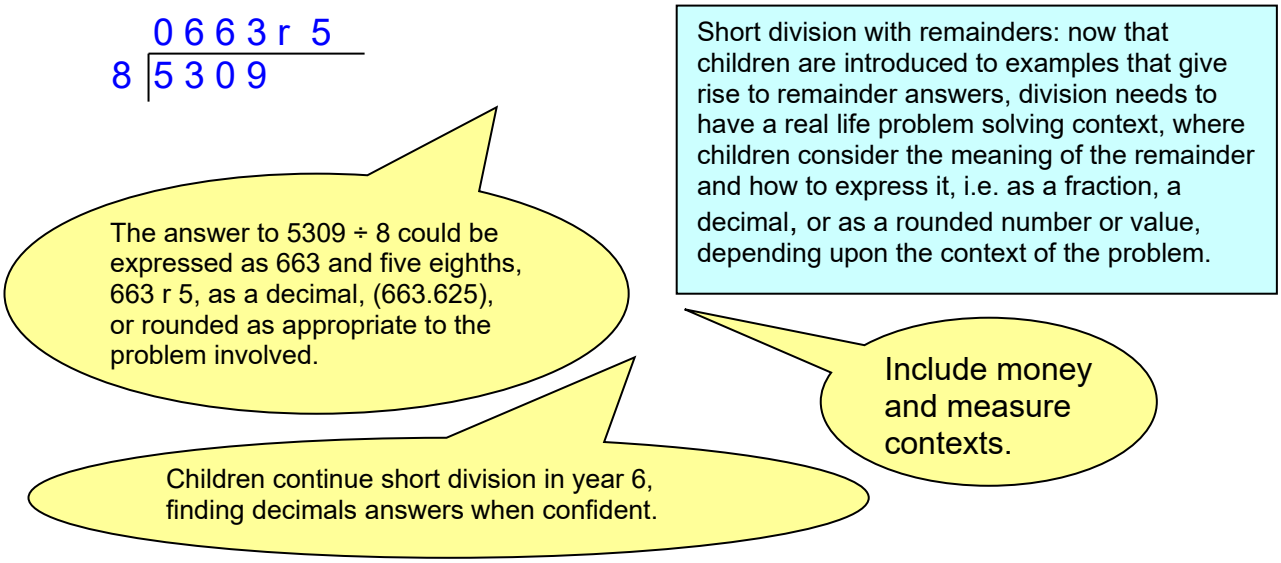
Non-Negotiables for multiplication in Stage Five:

- Identify multiples and factors using knowledge of multiplication tables to 12 x 12.
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

# Division Stage Five

**Divide up to four digits by a single digit, including those with remainders**

Short division, including remainder answers:




If children are confident and accurate:

- Introduce **long division** for pupils who are ready to divide any number by a two digit number (e.g. 2678 ÷ 19).

**Key vocabulary:** share, share equally, one each, two each … group, groups of, lots of, array, divide, divide by, divided into, division, grouping, number line, left, left over, inverse, short division, 'exchange', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime)

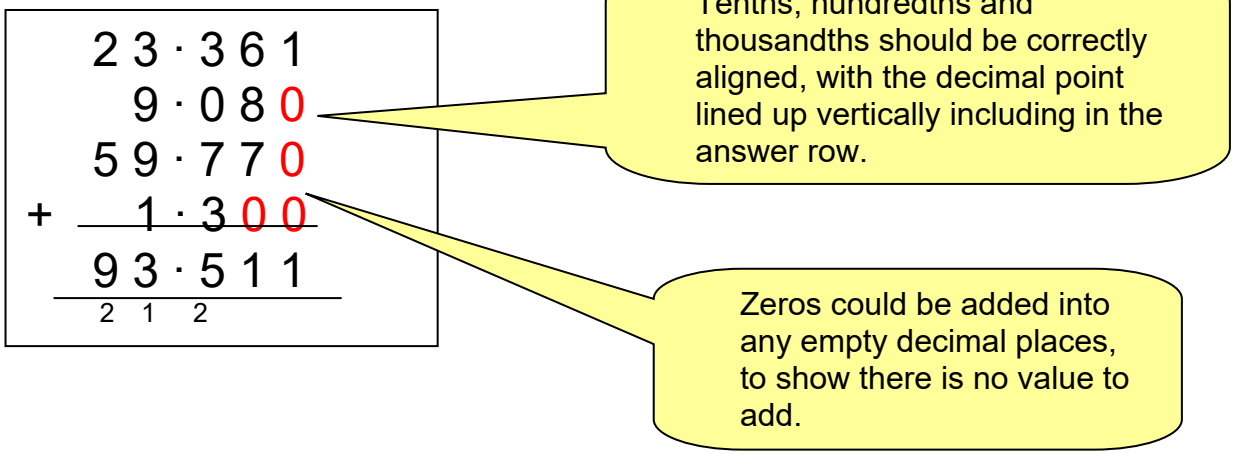
Key number skills needed for division at Stage Five:

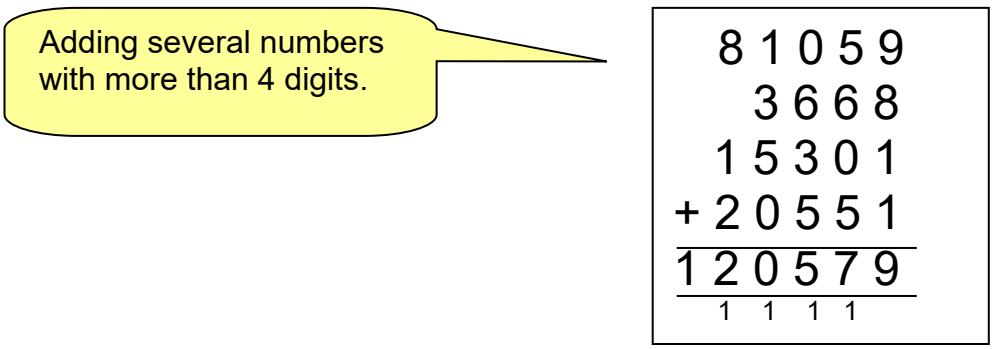
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Year Four)
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to four digits by a one digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 24½ = 24.5 approx 25)
- Solve problems involving combinations of all four operations, including understanding of the equals sign and including division for scaling by different fractions and problems involving simple rates.

# Addition Stage Six

**Add numbers with more than four digits**
Add several numbers of increasing complexity.

Adding several numbers with different numbers of decimal places (including money and measures):




**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal place, decimal point, tenths, hundredths, thousandths.

Non-Negotiables for addition in Stage Six:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Understand how to add mentally with larger numbers and calculations of increasing complexity.

# Subtraction Stage Six

**Subtracting with increasingly large and more complex numbers and decimal values**

Empty decimal places can be filled with zero to show the place value in each column.

```
  150699
-  89949
  60750
```

Using the compact column method to subtract more complex integers.

```
  105· 419 kg
-  36· 080 kg
   69· 339 kg
```

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Children should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

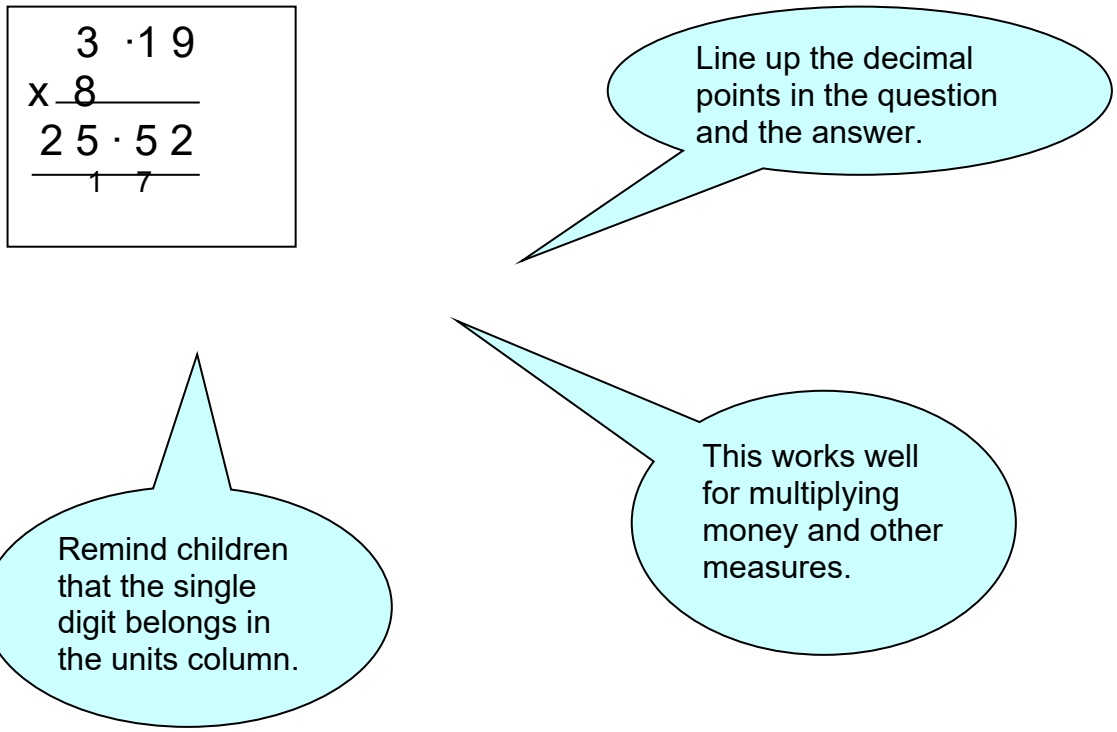
**Key vocabulary:** equal to, take away, less, minus, subtract, leaves, difference, between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is _ ?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Non-Negotiables for subtraction in Stage Six:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Use negative numbers in context and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

# Multiplication Stage Six

**Short and long multiplication, as in Year Five, and multiply decimals with up to two decimal points by a single digit.**

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**Children will be able to:**

- Use rounding and place value to make approximations before calculating and use these to check answers.
- Use short multiplication (see Year Five) to multiply numbers with **more than four digits by a single digit;** to multiply money and measures and to **multiply decimals with up to two decimal places by a single digit.**
- Use long multiplication (see Year Five) to multiply numbers with **at least four digits by a two digit number.**

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times … partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, 'exchange', tenths, hundredths, decimal.

Non-Negotiables for multiplication in Stage Six:

- Recall multiplication facts for all times tables up to 12 x 12 (as in Year Four and Five)
- Multiply multi-digit numbers, up to four digit, using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using rounding and approximation to determine levels of accuracy.
- Round any integer to a required degree of accuracy.

# Division Stage Six

**Divide at least four digits by both single digit and two digit numbers (including decimal numbers and quantities**

Short Division, for dividing by a single digit: e.g. 6497 ÷ 8

```
  0 8 1 2 · 1 2 5
8 6 4 9 7 · 0 0 0
```

Short Division with remainders: children should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where children have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: in this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is exchanged for the zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce Long division by subtracting multiples of the divisor (Chunking) for dividing 2 digits.

```
      2 7
3 6 | 9 7 2
    - 7 2 0      2 0 x
      2 5 2
    - 2 5 2      7 x
          0
    Answer:      2 7
```

Find out 'How many 36s are in 972?' by subtracting 'chunks' of 36 until 0 is reached (or until there is a remainder).

Children should be taught to write a 'useful list' first at the side that will help them decide what chunks to use, e.g.

'Useful list': 1 x = 36
10 x = 360
100 x = 3600

Introduce the method in a simple way by limiting the choice of chunks to 'can we use 10 lots?' 'Can we use 100 lots?' As children become more confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful lists'.

Where remainders occur, children should express them as fractions, decimals or use rounding, depending upon the problem.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times … partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, 'exchange', tenths, hundredths, decimal, common factor.

Non negotiables for division in Stage Six:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations.
- Divide numbers up to four digits by a two digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all four operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.